

The Bridge to A level Maths

Problem Solving OPTIONAL



West Coventry



Academy

Mathematics Department -
the department that counts!

1 Solving quadratic equations

Question 1

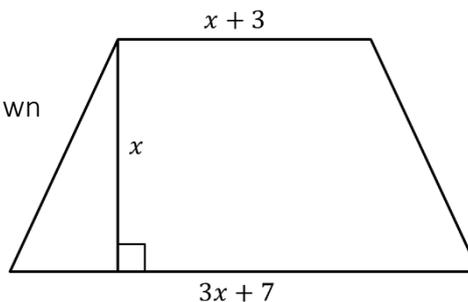
A number and its reciprocal add up to $\frac{26}{5}$.
Form and solve an equation to calculate the number.

.....
(3)

Question 2

The diagram shows a trapezium.

Diagram **NOT** accurately drawn



All the measurements are in centimetres.
The area of the trapezium is 16 cm^2 .

a) Show that $2x^2 + 5x - 16 = 0$

(1)

b) Work out the value of x to 1 decimal place.

$x = \dots\dots\dots$ (2)

Question 3

Two numbers have a product of 44 and a mean of 7.5.
Use an algebraic method to find the numbers.
You must show all of your working.

.....
(4)

Total / 10



2 Changing the subject

Question 1

The surface gravity of a planet is given by $g = \frac{GM}{r^2}$ where

M = Mass of the planet

r = radius of the planet

G = gravitational constant = 6.67×10^{-11}

The surface gravity of Earth is 9.807 m/s^2 and the mass of Earth is $5.98 \times 10^{24} \text{ kg}$.

Find the radius of Earth in kilometres correct to 3 significant figures.

(4)

Question 2

In a parallel circuit, the total resistance is given by the formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Make R_1 the subject of the formula

(4)

Question 3

Show that $\frac{1}{\frac{1}{x} + 1} = \frac{x}{x+1}$

(2)

Total / 10

Question 1

Sarah intended to spend £6.00 on prizes for her class but each prize cost her 10p more than expected, so she had to buy 5 fewer prizes.

Calculate the cost of each prize.

.....
(5)**Question 2**

Arthur and Florence are going to the theatre.

Arthur buys 6 adult tickets and 2 child tickets and pays £39.

Florence buys 5 adult tickets and 3 child tickets and pays £36.50.

Work out the costs of both adult and child tickets.

.....
(5)

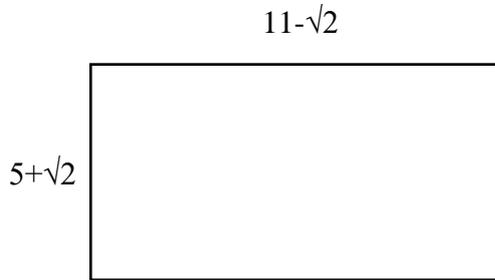
Total / 10

4 Surds

Question 1

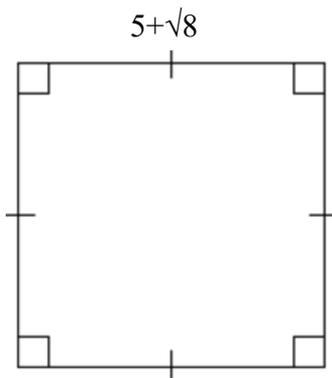
Calculate the area of each shape giving your answers in the form $a + b\sqrt{2}$

a)



(2)

b)



(3)

Question 2

Colin has made several mistakes in his 'simplifying surds' homework. Explain his error and give the correct answer.

i) $4\sqrt{3} \times 5\sqrt{12} = 20\sqrt{36}$

(2)

Question 3

The area of a triangle is 20cm^2 . The length of the base is $\sqrt{8}\text{cm}$. Work out the perpendicular height giving your answer as a surd in its simplest form.

(3)



Total / 10

Question 1

Lowenna says that $27^{-\frac{1}{3}} \times 64^{\frac{2}{3}} = 48$

Is Lowenna correct? You must show all of your working.

(4)

Question 2

Which one of these indices is the odd one out? Circle your answer and give reasons for your choice.

$$16^{-\frac{1}{4}}$$

$$64^{\frac{1}{2}}$$

$$8^{-\frac{1}{3}}$$

(2)

Question 3

Find values for a and b that make this equation work

$$a^{\frac{1}{2}} = b^{\frac{1}{3}}$$

(1)

Question 4

i) Write 25 as a power of 125

(1)

ii) Write 4 as a power of 32

(1)

iii) Write 81 as a power of 27

(1)

Total / 10

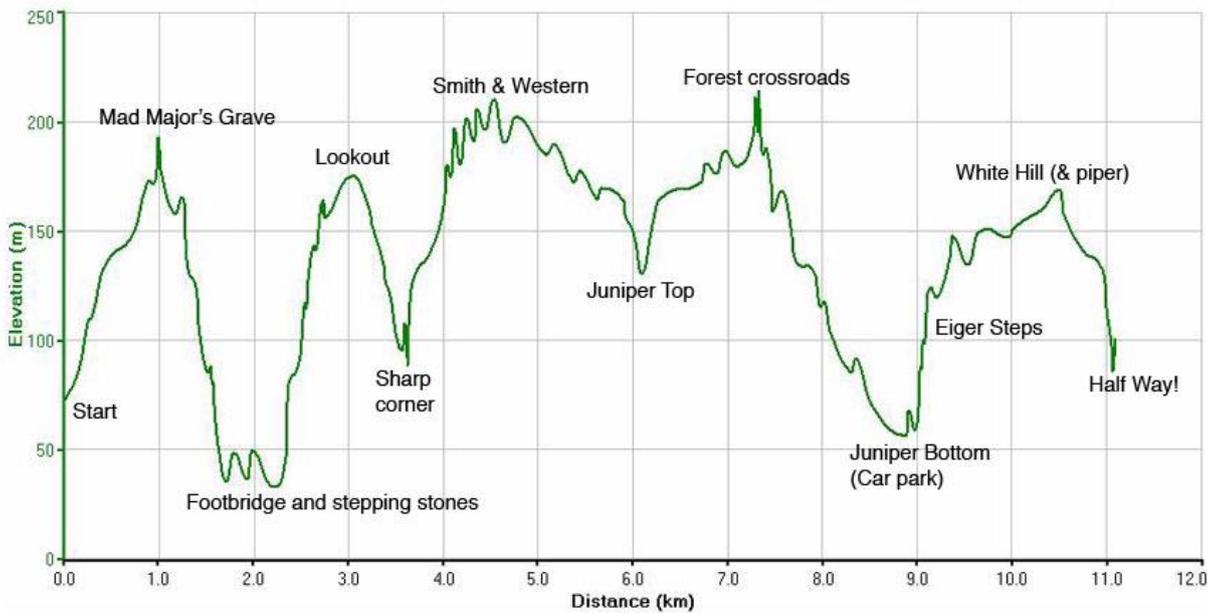
6 Properties of Lines

Question 1

- (a) Write down the gradient of the line $2y - 4x = 5$. (1)
- (b) Write down the equation of a line parallel to $3y = 7 - 4x$. (1)
- (c) Write down the equation of a line with gradient $\frac{1}{2}$ and y-intercept of 6. (1)

Question 2

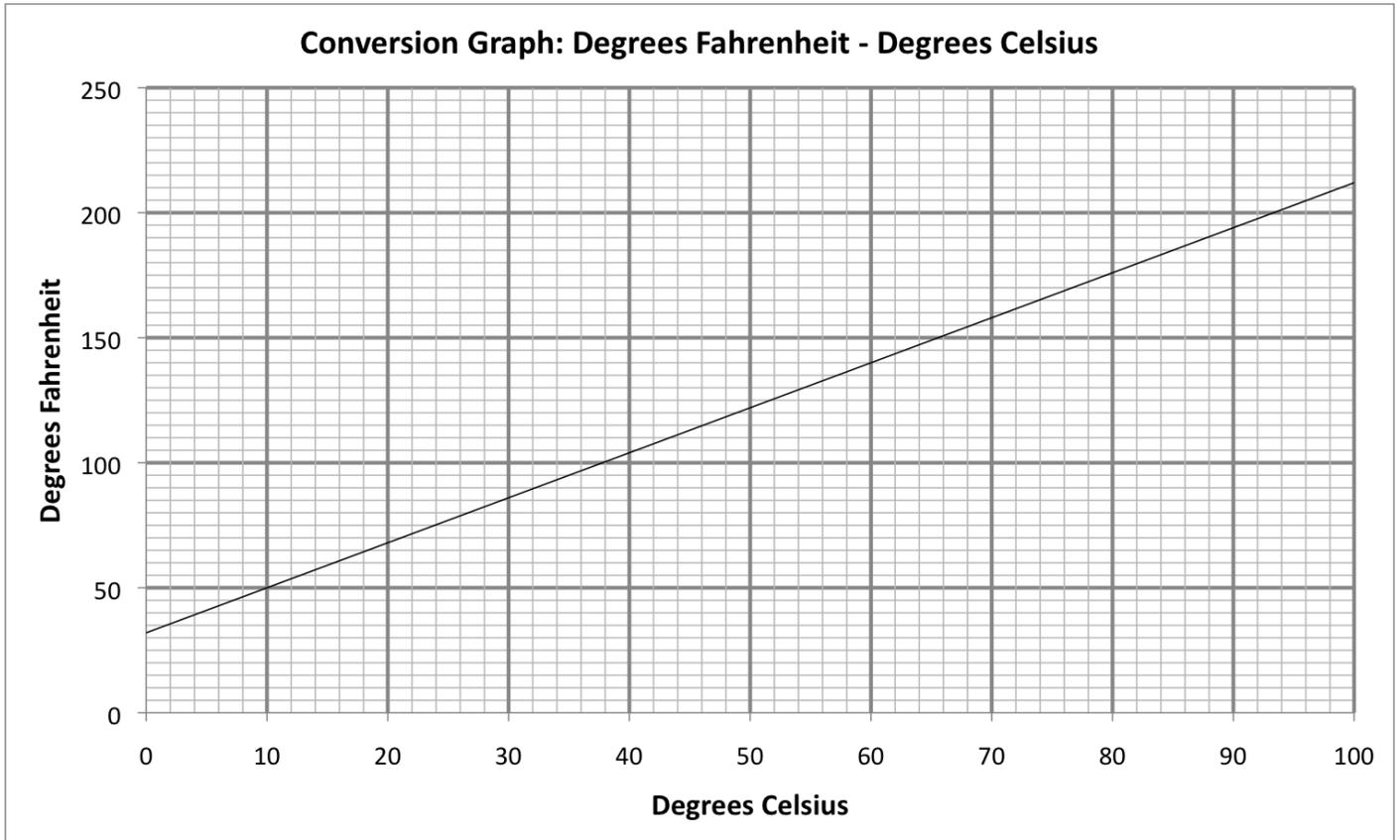
Here is the profile of the first half of a fell running race.



- (a) Work out the approximate gradient of the race from the start to Mad Major's Grave (2)
- (b) The most dangerous part of the race is from Mad Major's Grave to the Footbridge. Why do you think this might be? (1)
- (c) Work out an estimate for the average ascent for the first four uphill sections of the race. (2)

Question 3

Here is a graph used to convert degrees Celsius (C) and degrees Fahrenheit (F).



The equation of the straight line is given by $F = mC + a$
Calculate the values of m and a

(2)

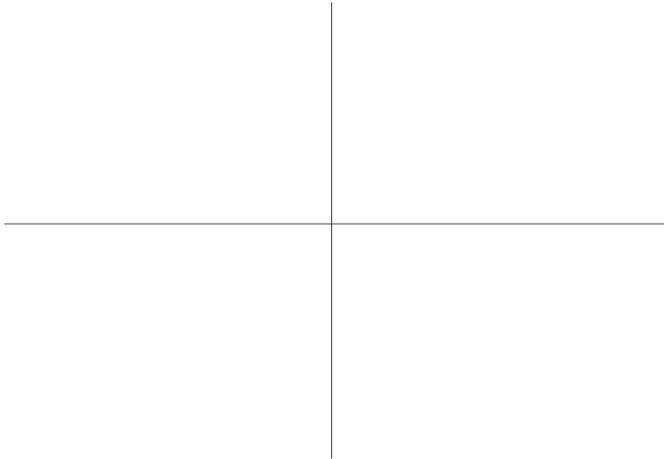
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7

Sketching curves

Question 1

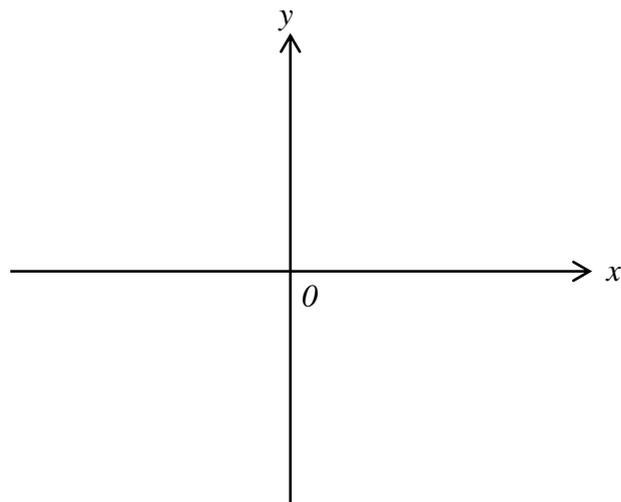
Sketch the graph of $f(x) = x^2 + 5x - 6$, showing the co-ordinates of the turning point and the coordinates of any intercepts with the coordinate axes.



(5)

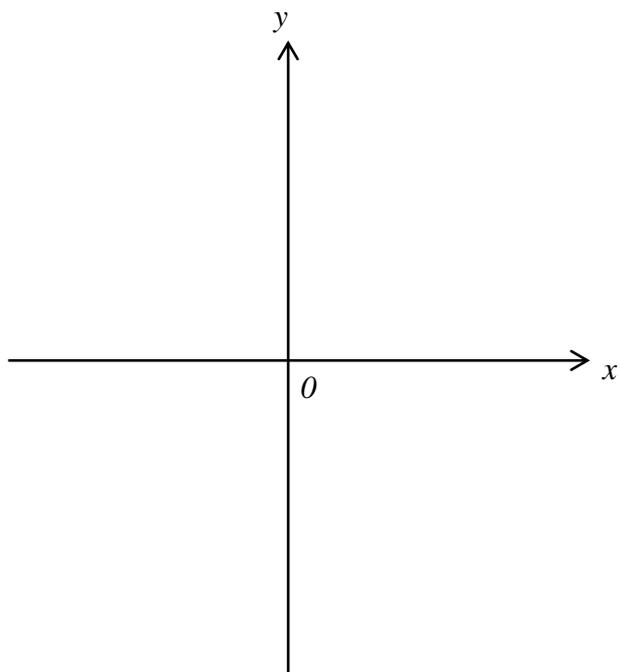
Question 2

- a) On the axes sketch the graph of $y = \frac{3}{x}$ showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.



(2)

b) On the axes sketch the graph of $y = x^3 - 5$ showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.



(3)

Total / 10

8 Transformation of functions

Question 1

Here is a sketch of $f(x)$.

The coordinates of P are $(0, -2)$

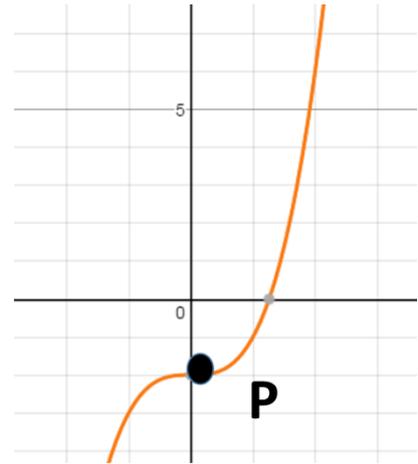
Sketch the graphs after the following translations and reflections, and state the coordinates of P' :

a) $g(x) = f(x) + 1$

b) $h(x) = f(x - 2)$

c) $j(x) = -f(x)$

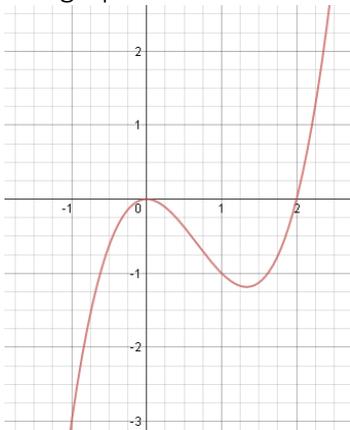
d) $k(x) = f(-x)$



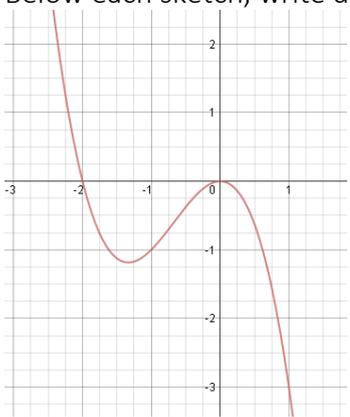
(4)

Question 2

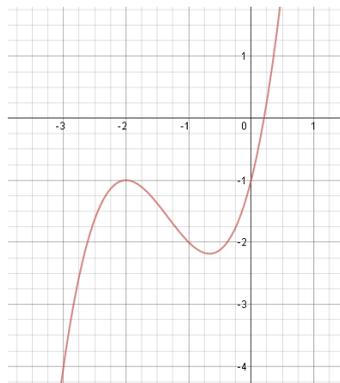
The graph of $y = f(x)$ is shown below.



Below each sketch, write down the equation of the transformed graph



$y = \dots\dots\dots$



$y = \dots\dots\dots$

(2)

Question 3

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 4x + 5$

C is the minimum point of the curve.

(a) Find the coordinates of C after the transformation $f(x + 1) + 2$.

(.....,)

(2)

(b) Hence, or otherwise, determine if $f(x - 3) - 1 = 0$ has any real roots.

Give reasons for your answer.

(2)

Total / 10

9 Pythagoras' theorem and Trigonometric ratios

Question 1

ABCDEFGH is a cuboid
 AE = 5cm
 AB = 6cm
 BC = 9cm

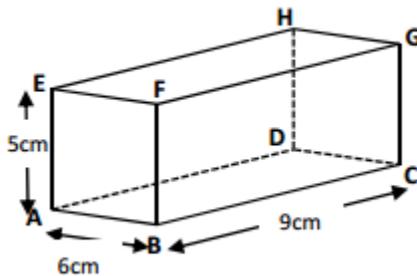


Diagram NOT drawn accurately

(a) Calculate the length of AG. Give your answer correct to 3 significant figures.

.....
 (1)

(b) Calculate the size of the angle between AG and the face ABCD.
 Give your answer correct to 1 decimal place.

(3)

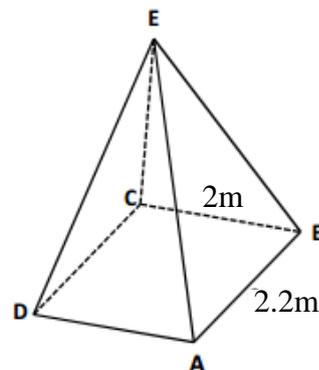
Question 2

A piece of land is the shape of an isosceles triangle with sides 7.5m, 7.5m and 11m.
 Turf can be bought for £11.99 per 5m² roll.
 How much will it cost to turf the piece of land?

.....
 (3)

Question 3

Ben is 1.62m tall.
 The tent he is considering buying is a square based pyramid.
 The length of the base is 3.2m.
 The poles AE, CE, BE and DE are 2m long.



Ben wants to know if he will be able to stand up in the middle of the tent. Explain your answer clearly.

.....
 (3)

Total / 10



Question 1

Plane A is flying directly toward the airport which is 20 miles away. The pilot notice a second plane, B, 45° to her right. Plane B is also flying directly towards the airport. The pilot of plane B calculates that plane A is 50° to his left. Based on that information how far is plane B from the airport? Give your answer to 3 significant figures.

.....
(4)**Question 2**

Two ships, A and B, leave the same port at the same time.

Ship A travels at 35km/h on a bearing of 130° .

Ship B travels at 25km/h on a bearing of 120° .

Calculate how far apart the ships are after 1 hour.

Give your answer correct to two decimal places.

.....
(3)**Question 3**

A farmer has a triangular field. He knows one side measures 450m and another 320m. The angle between these two sides measures 80° . The farmer wishes to use a fertiliser that costs £3.95 per container which covers 1500m^2 . How much will it cost to use the fertiliser on this field?

(3)

Total / 10



11 Inequalities

Question 1

A new cylindrical tube of snacks is being designed so that its height is 3 times its radius and its volume must be less than 20 times its radius.

Create an inequality and find possible values for the radius.

.....
(3)

Question 2

A base jumper is going to jump off a cliff that is **50m** tall, the distance she travels downwards is given by the equation

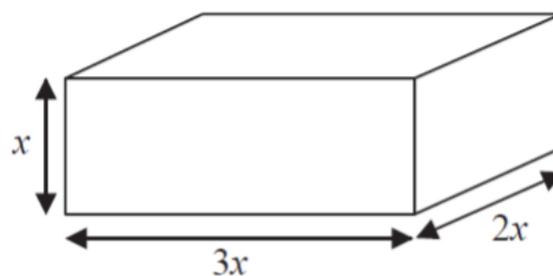
$$d = 4.9t^2 \quad \text{where} \quad t = \text{time of flight}$$
$$\text{and} \quad d = \text{distance travelled}$$

A video camera is set-up to film her between **20m** and **10m** above the ground. Calculate the time period after the jumper jumps that filming taking place.

.....
(3)

Question 3

The total volume of the box is less than 1 litre.
Given that all lengths are in cm and that x is an integer,
Show that the longest side is less than 18cm.



(4)

Total / 10



12 Algebraic proof

Question 1

Katie chooses a two-digit number, reverses the digits, and subtracts the smaller number from the larger.

For example

$$42 - 24 = 18$$

She tries several different numbers and finds the answer is never a prime number.

Prove that Katie can never get an answer that is a prime number.

(4)

Question 2

Here are the first 5 terms of an arithmetic sequence

$$1 \quad 6 \quad 11 \quad 16 \quad 21$$

Prove that the difference between the squares of any 2 terms is always a multiple of 5.

(6)

Total / 10

13 Vectors

Question 1

OAB is a triangle

$\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$

(a) Find the vector \vec{AB} in terms of \mathbf{a} and \mathbf{b}

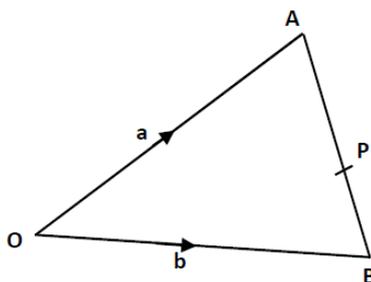


Diagram NOT drawn accurately

P is the point on \vec{AB} such that AP:PB = 3:2

(b) Show that $\vec{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

..... (1)

..... (3)

Question 2

OABC is a parallelogram.

X is the midpoint of OB

$\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$

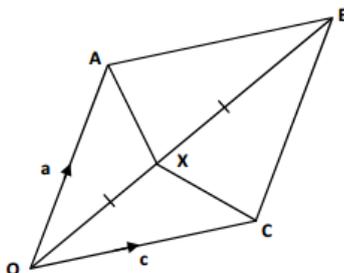


Diagram NOT drawn accurately

(a) Find the vector \vec{OX} in terms of \mathbf{a} and \mathbf{c} .

..... (1)

(b) Find the vector \vec{XC} in terms of \mathbf{a} and \mathbf{c} .

..... (2)

Question 3

PQRS is a parallelogram.
M is the midpoint of RS
N is the midpoint of QR

$$\vec{PQ} = 2\mathbf{a}$$

$$\vec{PS} = 2\mathbf{b}$$

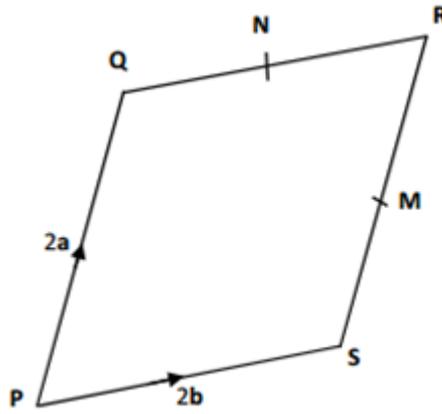


Diagram NOT
drawn
accurately

Use vectors to prove that the line segments SQ and MN are parallel.

(3)

Total / 10

14 Probability

Question 1

Max has an empty box.

He puts some red counters and some blue counters into the box.

The ratio of the number of red counters to the number of blue counters is 1 : 3.

Julie takes at random 2 counters from the box.

The probability that she takes 2 red counters is $\frac{19}{316}$.

How many red counters did Max put in the box?

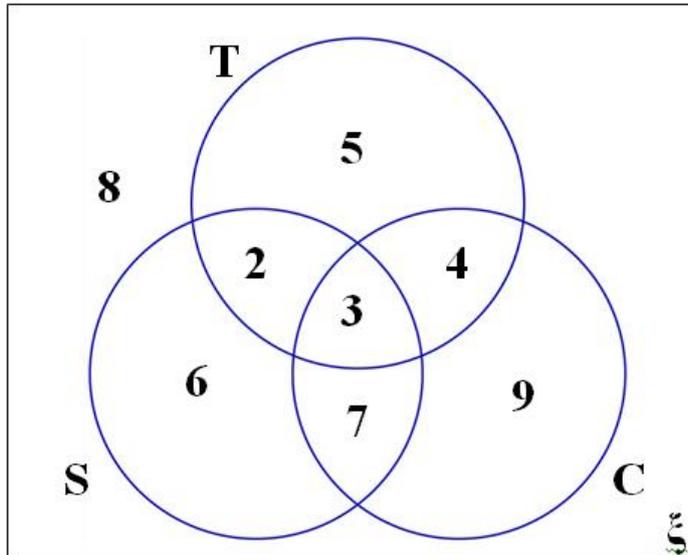
(5)

Question 2

The Venn diagram shows the ice-cream flavours chosen by a group of 44 children at a party.

The choices are strawberry (S), choc-chip (C) and toffee (T).

A child is picked at random.



Work out :

(a) $P(S)$

.....
(1)

(b) $P(T \cup C | C)$

.....
(2)

(c) $P(C | S \cup T)$

.....
(2)

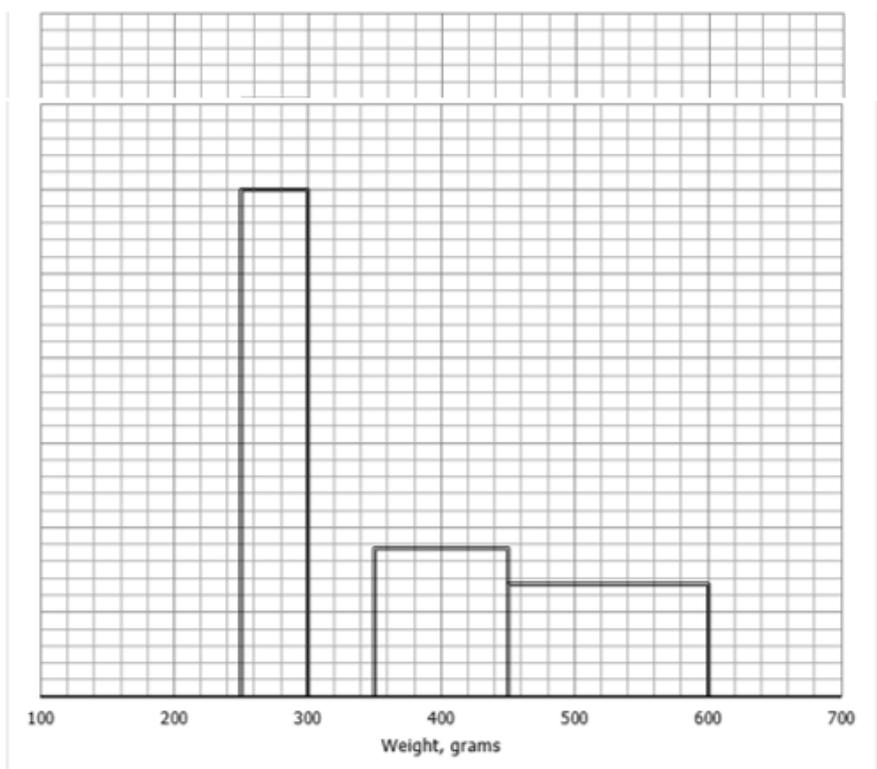
Total / 10



Question 1

The table and histogram show the weights of some snakes.

Weight, grams	Frequency
250 < x ≤ 300	60
300 < x ≤ 325	25
325 < x ≤ 350	40
350 < x ≤ 450	35
450 < x ≤ 600	40
Total	200



(a) Use the information to complete the histogram

(3)

(b) Calculate an estimate for the median

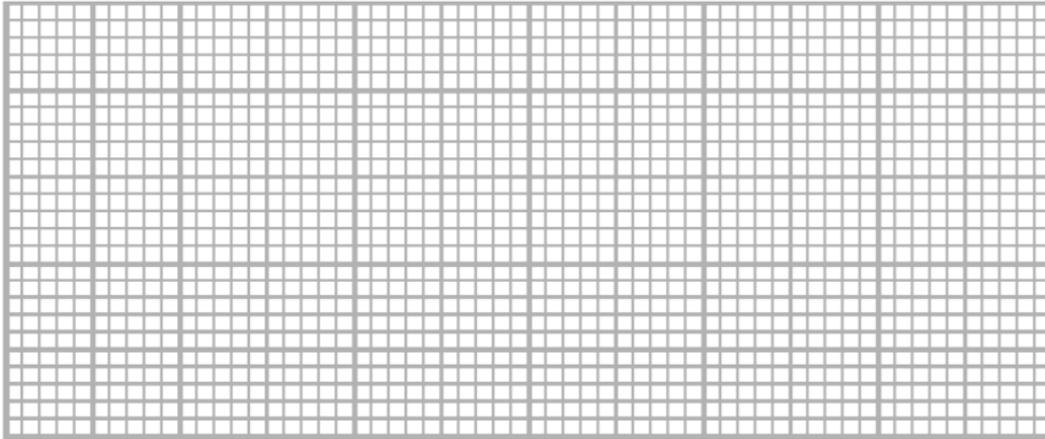
.....
(2)

Question 2

Sarah played 15 games of netball. Here are the number of goals she scored in each game.

17 17 17 18 19 20 21 22 24 25 25 26 28 28 28

a) Draw a boxplot to show this information



(3)

Lucy plays in the same 15 games of netball. The median number of points Lucy scores is 24. The interquartile range of these points is 10 and the range of these points is 17.

Who is the better player, Sarah or Lucy?
You must give a reason for your answer.

(2)

Total / 10